Sr.No. 14812

Exam.Code: 211004
Subject Code : 4640

M.Sc (Mathematics) - 4th Semester

(2720)

Paper : MATH-579

Advanced Numerical Analysis

Time Allowed: 2 hrs. **Max. Marks: 100**

Note: Attempt any four questions. All questions are of equal marks.

1(*a*). Given the points $(1,6),(2,18),$ *and* $(3,42)$ satisfying the function $y = x^3 + 5x$, determine the cubic spline in the interval $[1,2]$ using the end conditions $y'(1) = 8$ and $y'(3) = 32$. $12\frac{1}{2}$

(*b*). Obtain the first four orthogonal polynomials $f_n(x)$ on $[-1,1]$ with respect to the weight function $W(x) = 1$. $12\frac{1}{2}$

2(*a*). Define Fourier series of a function. Find the Fourier series of the function defined by

$$f(x) = \begin{cases} x, & -1 < x \le 0 \\ x+2, & 0 < x \le 1. \end{cases}$$ $12\frac{1}{2}$

(*b*). Define Chebychev Polynomail T_n of degree n in interval [-1,1] and prove that $T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x)$.

Further show that $T_n(x)$ satisfies the differential equation

$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + n^2y = 0.$ $12\frac{1}{2}$

3(*a*). Solve the Laplace equation $U_{xx} + U_{yy} = 0$ for the region bounded by the square $0 \le x, y \le 4$, the boundary conditions being

$u = 0 \; at \; x = 0 \; and \; u = 8 + 2y \; at \; x = 4$

$u = \frac{1}{2}x^2 \; at \; y = 0 \; and \; u = x^2 \; at \; y = 4.$

With $h = k = 1.0$, use Gauss-Seidel's method to compute u at the internal mesh points. $12\frac{1}{2}$

PTO

(*b*). Solve the Laplace equation with $h = 1/3$ over the boundary of a square of unit length with $u = 9x^2y^2$ on the boundary. 12½

4(*a*). Using Crank- Nicolson method solve the equation $U_{xx} = 2U_t$;

$$U(0,t) = U(4,t) = 0 \text{ and } u(x,0) = x(4-x).$$

Taking $h = 1,\ r = 1,$ *find values up to third level.* 12½

(*b*). Solve explicitly, the wave equation

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$$

with boundary conditions $u(0,t) = 0,\ u(1,t) = 0,\ t > 0$ and the initial conditions $u(x,0) = sin^3(\pi x),\ \frac{\partial u}{\partial t} = 0$ *when* $t = 0,\ 0 \le x \le 1$

forh $= 0.25$ *and* $k = 0.2$. 12½

5(*a*). Solve $y'' + y = 3x^2,\ y(0) = 0\ \&\ y(2) = 3.5$, by Galerkin method.

Take trial solution $u(x) = \frac{7x}{4} + c_2(x)(x-2) + c_3x^2(x-2)$. 12½

(*b*). Solve the boundary value problem $y'' + y = -x,\ 0 < x < 1$ with

$y(0) = \quad y(1) = 0$, by Rayleigh-Ritz method. 12½

6(*a*). Apply Collocation method to solve

$$y'' + xy = x^3 - \frac{4}{x^3};\ y(1) = -1, \quad y(2) = 3.$$

Take $x = 1.25, 1.5$ *and* 1.75 as the interior collocation points. 12½

(*b*). By using the Finite Element Method, solve the problem

$y''(x) + 2 = 0,\ 0 < x < 1,\ y(0) = 0 = y'(1)$. Take two elements of the given interval. 12½

Contd...P/3

$7(a)$. Let X be a random variate distributed with the probability density function (pdf) $f_X(x), x \in I$, which is represented as $f_X(x) = Cg(x)h(x)$, where $C \geq 1, 0 < g(x) \leq 1$ and $h(x)$ is also pdf. Let U and Y be distributed as $U(0,1)$ and $h(x)$, respectively. Then $f_Y\left(x \middle| U < g(Y)\right) = f_X(x)$. 10

(b) Describe inverse transform method to generate a random number from probability density function $f(x)$. Hence, write the algorithm for generating n random numbers from a exponential distribution. 8.75

(c) Discuss a method to generate a random number from a Poisson distribution. 6.25

$8(a)$. Consider the integral $I = \int_a^b g(x)dx$, where $0 \leq g(x) \leq c$, for $a < x < b$. Let θ_1 and θ_2 be the estimators of I using Hit or Miss Method and Sample Mean Monte Carlo method, respectively. Prove that $Var(\theta_2) \leq Var(\theta_1)$, for equal number of simulation runs N. 10

(b) Describe any one variance reduction technique in Sample Mean Monte Carlo Method. 8.75

(c) Explain Monte Carlo method to estimate π using Buffon's Needle Problem. 6.25

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